S.2 Mathematics | ALGEBRA

Algebraic expressions and Equations

Algebraic expressions are expressions that involve letters or symbols. Numbers can be represented by letters or symbols and what is formed in algebraic expressions. E.g.

a+3, 2n, 3ab, 2a+5 etc.

These expressions are treated like it were a number. We can add, subtract, multiply and divide like we do for numbers.

Algebraic equations

We can use algebraic expressions to form algebraic equations. e.g.

a + 3 = 5

2n = 12

3x + 5 =17

2a + b =36

Example 1:

Form an algebraic equations for each of the statements below.

a) John is twice as old as his son James.

b) My father gave me half his books. I had 10 books already but now I have 35 note books.

c) I have 5 more pens than pencils.

d) If had 5 more cows than I have now, I should have 9.

Solutions

a) Let John's age be y and James' age be x

$$y = 2x$$

b)Let the number of books my father had be n, he gave $\frac{n}{2}$

$$10 + \frac{n}{2} = 35$$

c) Let the number of pencils I have be a and pens be b

$a + 5 = b$ or $b - a = 5$ or $b - 5 = a$

d) Let the number of cows I have now be n

$$n + 5 = 9$$

Example 2:

Simplify the following algebraic expressions

a) 2a + 3 + 5a
b) 13 – b + 6b
c) 3 (1 + a) – 4a

Solutions

a) 2a + 3 + 5a

Arrange the terms containing a together and collect like terms.

2a + 5a + 5

7a + 5

b) 13 – b + 6b

13 + 6b – b

13 + 5b

c) 3 (1 + a) – 4a

3 x 1 + 3 x a – 4a

3 + 3a – 4a

3 – a

Example 3:

Simplify these expressions

a) 2p – 5 + 7p
b) 2 (x + 9) – 3X + 4
C) 2xy – 1 + 4xy
d) xyz + yzx + zxy+ xzy – 3xyz
e) ab + 2bc – 3ac + 1

Solutions

a) 2p – 5 + 7p

= 2p + 7p – 5

= 9p – 5

b) 2 (x + 9) – 3x + 4

2x + 18 – 3x + 4

2x – 3x + 18 + 4

-x + 22 or 22 – x

c) 2xy – 1 + 4xy
2xy + 4xy – 1
6xy – 1

d) xyz + xyz + xyz + xyz – 3xyz
=4xyz – 3xyz
=xyz

e) ab + 2abc – 3ac + 1
The terms in part (e) do not have exactly the same letters; therefore, they are not like terms and cannot be simplified.

Example 4:

(Substitution)
a) Given that, a= 2b – 3c, find the value of a when
i) b = 12, c = 5
ii) b = 9, c=1
iii) b = 8, c = 6

b) m = n^2+ $\frac{1}{4}$p. Find the value of m when
i) n = 4, p = 8
ii) n = 6, p = 4
iii) n = 20, p = 400

Solutions

a) a = 2b – 3c
i) when b = 12, c = 5
a = 2(12) – 3(5)
= 24 – 15
= 9

ii) When b = 9, c = 1
a = 2(9) – 3(1)
= 18 – 3
= 15
iii) When b =8, c = 6
a = 2(8) – 3(6)
= 16 p – 18
= -2

b) m = $n^2 + \frac{1}{4}p$

i) when n= 4, p = 8

m = $(4)^2 + \frac{1}{4}(8)$

= 16 + 2

= 18

ii) When n=6, p=4

m= $(6)^2 + \frac{1}{4}(4)$

= 36 + 1

= 37

iii) When n = 20, p= 400

m= $20^2 - \frac{1}{4}(400)$

= 400 – 100

= 300

EXERCISE

1) Form an algebraic expression for each of the following quantities.

a) I have n bananas, my sister has three more than I do. How many bananas does she have?

b) I weigh ykg. I am 7kg lighter than my mother. How many kg does my mother weigh?

c) Mariam's rope is bcm long and Milly's is 8cm longer. How long is Milly's rope in terms of Mariam's?

d) I am f years old. My brother is twice as old as I am. How old is my brother?

2) Form an algebraic equation for each of these statements.

a) Yesterday I picked m bags of oranges. Today, I picked n bags. Altogether I picked 25 bags.

b) I have h hens. If I had six more, I should have 18 hens.

c) There were ten trees in the field. A ranger cut down t of them. How many are left?

d) My sister has a total of 3m beads. She has 12 white onesand n blue ones.

3. A pen costs **b** shillings and a pencil costs 900 shillings less than a pen. **Write an expression for the total cost of a pen and a pencil.**

4. Simplify the following by grouping the positives and negatives.

a) 11a + 5a – 2a + a

b) 12d – 5d – 3d + 4d

c) 17p – 8p –2p

d) 4b + 2b – 9b + 8b

5. Simplify the following by collecting like term

a) 15 – 2x + 10x

b) 4y – 2x + 5x – 3y

c) 20x – 4y – y – 3x

d) 9a + 10b – 5a – 4b

Algebraic products

If an expression contains two or more terms, we may write it in brackets. E.g. 4 +3 may be written as (4+3) or a + b + c may be written as (a + b + c). If multiply this number by a number say 2, itmultiplies each term inside the bracket.

i.e. 2(4 + 3) = 2x4 + 2x3

=8 + 6

=14

But 2 (4 + 3) is also equal to

2 x 7

=14

Similarly 2(a + b + c)

Example 1:

Multiply out i) 5(2a + 4b)

ii) 32(2w + 3x – 4y)

iii) (2m – 5n) 2l + (3l + 4m) 2n

Solution

i) = 5(2a + 4b)

= 5 x 2a + 5 x 4b

= 10a + 20b

ii) 3x (2w + 3x – 4y)

= 3x x 2w + 3x x 3x – 3x x 4y

= 6xw + $9x^2$ – 12xy

iii) (2m – 5n) 2l + (3l + 4m) 2n

= 4ml – 10ln + 6ln + 8mn
= 4ml – 4ln + 8mn

Note that 5(2a – 4b) = (2a – 4b)5

The number outside the brackets is called a factor.

Example 2:
Evaluate each expression in two ways

a) 3(6 + 3)
b) 4(7 + 4)
c) 6(9 – 7)
d) (2 + 5) x 8
e) (6 + 5) x 7
f) (11 – 6) x 6

Solution

a) 3(6 + 3) = 3 x 6 + 3 x 3
= 18 + 9
= 27
OR 3(6 + 3) = 3 x 9
= 27

b) 4(7 + 4) = 4 x 7 + 4 x 4
= 28 + 16
= 44
OR 4(7 + 4) = 4 X 11
= 44

c) 6(9 – 7) = 6 x 9 – 6 x7
= 54 – 42
= 12
OR 6(9 – 7) = 6 x 2
= 12

d) (2 + 5) x 8 = 2 x 8 + 5 x 8
= 16 + 40
= 56
OR (2 + 5) x 8 = 7 x 8
= 56

e) (6 + 5) x 7 = 6 x 7 + 5 x 7
= 42 + 35
= 77

OR (6 + 5) x 7 = 11 x 7
= 77

f) (11 – 6) x 6 = 11 x 6 – 6 x 6
= 66 – 36
= 30
OR (11 – 6) x 6 = 5 x 6
= 30

Example 3:
Multiply each term in the brackets by the factor.
a) 4(a + b)
b) 2(3g + h)
c) 3(3n – 2)
d) 4(r – 2s + 3t)
e) (5n – 3) p
f) 3a(b + 2c – 4d)

Solution
a) 4(a + b) = 4 x a + 4 x b = 4a + 4b

b) 2(3g + h) = 2 x 3g + 2 x h = 6g + 2h

c) 3(3n – 2) = 3 x 3n – 3 x 2 = 9n – 6

d) 4(r – 2s + 3t) = 4 x r – 4 x 2s + 4 x 3t = 4r – 8s + 12t

e) (5n - 3)p = 5n x p – 3 x p = 5np – 3p

f) 3a(b + 2c – 4d) = 3a x b + 3a x 2c – 3a x 4d
= 3ab + 6ac – 12ad

Example 4:
Expand and simplify each of the following expressions as much as possible
a) 2(a + b) + 3(a – b)
b) 4(2c + d) + 5(d – c)
c) 5(3j – k) – 4(5k + m)
d) 2g(h – 2j) – 3g(h – 2j)

Remember
*When there is a positive sign before the bracket, the signs of the terms inside the bracket remain the same when the bracket are removed.

*When there is negative sign before the bracket, the signs of the terms inside the brackets change when the brackets are removed.

Solutions

a) = 2(a + b) + 3(a – b)
= 2a + 2b + 3a – 3b
= 2a + 3a + 2b – 3b
= 5a – b

b) = 4(2c + d) + 5(d – c)
= 8c + 4d + 5d – 5c
= 8c – 5c + 4d + 5d
= 3c + 9d

c) = 5(3j – k) – 4(5k + m)
= 15j – 5k – 20k – 4m
= 15j – 25k – 4m

d) = 2g(h – 2j) – 3g(h – 2j)
= 2gh – 4gh – 3gh + 6gj
= 2gj – gh

EXERCISE

1) Remove the brackets and then simplify.
a) 6x + (3 – x)
b) (2p + 6q) – q
c) (x – 3y) + (x + 3y)
d) (12a + 7) + (4 – 5a)

2) Remove the brackets from the following (or expand).
a) 6(7 - h)
b) 7(2x + 9y)
c) 3x(2y – 7)
d) (w – x)4x

3) Simplify the expressions as much as possible.
a) 2(3e + 2f) + 5(e – 2f)
b) 2(3p + q) + 3(p – 2q)
c) 3(4g – 3h) – 2(5g + 2h)
d) 2d(e – 3f) – d(e – 2f)

Binomial Products

A binomial is an expression with two terms. E.g.a + b, 2x + 3, 2 + a, etc.

An expression with two or more terms may be written with brackets as (a + b), (2x + 3), (2 + aS).